

## Abstract

The post-LEP  $e^+e^-$  colliders proposed for the precise high-energy physics will provide a set of ground-breaking measurements of a large number of new-physics sensitive electroweak pseudo-observables (EWPOs), with improvement by one to two orders of magnitude in experimental precision. The full exploitation of the significantly increased experimental precision at the Z boson resonance region for EWPOs (effective weak mixing angles, the Z-boson partial and total decay widths, the branching ratios, the hadronic cross section), necessitates Standard Model (SM) predictions accurate at a level commensurate with this precision, demanding leap-jumps in the precision of higher-order perturbation calculation represented by multi-scale multi-loop Feynman integrals. We discuss different techniques used for the evaluation of these Feynman integrals beyond the one-loop level, focusing on Mellin-Barnes representations, sector decomposition and differential equations. In this respect, we developed auxiliary programs and procedures to automate calculations with sector decomposition and differential equation methods. These methods have been used further to calculate SM three-loop  $W$  and  $Z$  boson self-energies and the  $W\bar{l}\nu_l$  vertex of the order  $\mathcal{O}(\alpha^2\alpha_s)$ . These corrections are missed so far and are needed for a full exploration of EWPOs, in particular for the determination of the  $\rho$  parameter and the muon decay  $\Delta r$  parameter. We also show that with present tools and methods, it is possible to calculate numerically with sufficient precision the most difficult vertex integrals in Minkowskian kinematics, required by future collider physics analysis and needed for the evaluation of three-loop SM corrections to the Z boson decays.