Applications of topos theory in quantum physics
Abstract

Krzysztof Bielas

This thesis explores various applications of topoi in the realm of quantum physics. In particular, the work employs a method of variable mathematical foundations, wherein the conventional topos **Set** is consequently substituted with sheaf topoi $Sh(B)$ (derived from Boolean-valued models V^B built upon Boolean algebras of projections on a Hilbert space). On occasion, the Basel topos $\mathcal{B}$ is also utilized. It is argued that the approach may shed new light on the cosmological constant problem by altering the structure of real line and, consequently, the smooth structure of spacetime. Furthermore, the possible connection with exotic smooth structures on $\mathbb{R}^4$ is discussed. Eventually, the problem of randomness of quantum mechanics is addressed and it is demonstrated that a quantum system described by the infinite-dimensional Hilbert space formally exhibits a stronger notion of randomness.

Keywords — quantum mechanics, topoi, Boolean-valued models, exotic smoothness, randomness